Important Concepts

### Exponential Growth

An exponential pattern of change can often be recognized in a verbal description of a situation or in the pattern of change in a table of \((x, y)\) values.

The exponential growth in rewards for good-work days in the example can be represented in a graph. The increasing rate of growth is reflected in the upward curve of the plotted points.

### Growth Factor

A constant factor can be obtained by dividing each successive \(y\)-value by the previous \(y\)-value. This ratio is called the growth factor of the pattern.

For each good-work day, the reward doubles. You multiply the previous award by 2 to get the new reward. This constant factor can also be obtained by dividing successive \(y\)-values:

\[
\frac{2}{1} = 2, \quad \frac{4}{2} = 2, \quad \text{etc.}
\]

On the \(n\)th day, the reward \(R\) will be \(R = 1 \times 2^n\). Because the independent variable in this pattern appears as an exponent, the growth pattern is called exponential. The growth factor is the base 2. The exponent \(n\) tells the number of times the 2 is a factor.

### Exponential Equation

Examining the growth pattern leads to a generalization that can be expressed as an equation.

An exponential growth pattern \(y = a(b)^x\) may increase slowly at first but grows at an increasing rate because its growth is multiplicative. The growth factor is \(b\).

### Exponential Decay

Exponential models describe patterns in which the value decreases. Decay factors result in decreasing relationships because they are less than 1.

By examining the multiplicative structure of the bases:

\[
8^2 = (2 \times 2)^2 = (2^3)^2 = 2^6; \quad \text{the general pattern is} \quad (b^m)^n = b^{mn}
\]
\[
9 \times 27 = 243 \quad \text{or} \quad 3^2 \times 3^3 = 3^5; \quad \text{in general,} \quad (b^m)(b^n) = b^{m+n}
\]
\[
4 \times 25 = 2^2 \times 5^2 = (2 \times 5)^2 = 10^2 = 100; \quad \text{in general,} \quad (a^m b^n) = (ab)^m
\]

Similar explorations lead to the rule \(\frac{a^m}{a^n} = a^{m-n}\).