Thinking With Mathematical Models

Investigation 1, ACE Exercise 5

Parts (a)–(f) refer to relationships between variables you have studied in this Investigation. Tell whether each relationship is **linear** or **nonlinear**.

a. Cost depends on truss length (ACE Exercise 3).

b. Cost depends on the number of rods in a staircase frame (ACE Exercise 3).

c. Bridge strength depends on bridge thickness (Problem 1.1).

d. Bridge strength depends on bridge length (Problem 1.2).

e. Number of rods depends on truss length (Problem 1.3).

f. Number of rods depends on the number of steps in a staircase frame (Problem 1.3).

g. Compare the patterns of change for all the nonlinear relationships in parts (a)–(f).

**HINT:**

1. For each of the relationships described in parts (a)–(f), look at the graphs you have previously made. How do you know a situation is linear from a graph?

2. For each of the relationships described in parts (a)–(f), look at the tables you have previously made. How do you know a situation is linear from a table?

3. Think about the pattern of change in each of the situations described in parts (a)–(f). What do you know about the patterns of change in linear situations?
Thinking With Mathematical Models

Investigation 2, ACE Exercise 5

The U-Wash-It car wash did market research to determine how much to charge for a car wash. The company made this table based on its findings.

<table>
<thead>
<tr>
<th>U-Wash-It Projections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price per Wash ($)</td>
</tr>
<tr>
<td>Customers Expected per Day</td>
</tr>
</tbody>
</table>

a. Graph the (price, expected customers) data. Draw a line that models the data pattern.

b. Write an equation in the form $y = mx + b$ for your graph. Explain what the values of $m$ and $b$ tell you about this situation.

c. Use your equation to find the number of customers expected for prices of $2.50, $7.50, and $12.50.

HINT:

1. From your graph model, find the beginning amount, that is, the $y$-intercept. Where does this number appear in the equation?

2. Use your graph model to find the change in the number of customers for every increase of $1 in cost. Should you indicate this change in the number of customers by a positive or negative number?

3. Where does the number that represents the amount of change appear in the equation?
Thinking With Mathematical Models

Investigation 3, ACE Exercise 10

The route for one day of a charity bike ride covers 50 miles. Individual participants ride this distance at different average speeds.

a. Make a table and a graph that show how the riding time changes as the average speed increases. Show speeds from 4 to 20 miles per hour in intervals of 4 miles per hour.

b. Write an equation for the relationship between the riding time \( t \) and average speed \( s \).

c. Tell how the riding time changes as the average speed increases from 4 to 8 miles per hour, from 8 to 12 miles per hour, and from 12 to 16 miles per hour.

d. How do the answers for part (c) show that the relationship between average speed and time is not linear?

HINT:

1. Describe how can you determine the time needed to travel 50 miles at a speed of 4 miles per hour.

2. Write an equation using the variable \( s \) to represent speed (4 miles per hour) and the variable \( t \) to represent time.

3. Use your equation to make a table showing how time changes as speed changes. Use speeds of 4, 8, 12, and 16 miles per hour.

4. Use this table to answer parts (c) and (d).

5. How can you determine whether or not a relationship is linear? Explain.
Thinking With Mathematical Models

Investigation 4, ACE Exercise 1

Use the table below. It shows the height and stride distance for 10 students.

For humans, walking is the most basic form of transportation. An average person is able to walk at a pace of about 3 miles per hour.

The distance a person covers in one step depends on their stride. To measure stride distance, measure from the heel of the first foot to the heel of that same foot on the next step.

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th>Stride Distance (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150.8</td>
<td>125.2</td>
</tr>
<tr>
<td>149.5</td>
<td>124.2</td>
</tr>
<tr>
<td>151.2</td>
<td>125.2</td>
</tr>
<tr>
<td>153.1</td>
<td>126.8</td>
</tr>
<tr>
<td>150.6</td>
<td>124.4</td>
</tr>
<tr>
<td>149.9</td>
<td>123.8</td>
</tr>
<tr>
<td>146.5</td>
<td>121.8</td>
</tr>
<tr>
<td>146.5</td>
<td>120.8</td>
</tr>
<tr>
<td>151.5</td>
<td>125.6</td>
</tr>
<tr>
<td>153.5</td>
<td>126.8</td>
</tr>
</tbody>
</table>

a. What is the median height of these students? Explain how you found the median.

b. What is the median stride distance of these students? Explain how you found the median.

c. What is the ratio of median height to median stride distance? Explain.

HINT:
1. List the students’ heights from shortest to tallest.
2. List the students’ stride distances from shortest to longest.
3. The median is the middle number. Since this data set has an even amount of data points, the median will be the average of the two middle numbers.
4. A ratio is a relationship between two numbers of the same kind. You can write this as median height to median stride or as a fraction.
Thinking With Mathematical Models

Investigation 5, ACE Exercise 15

You can analyze data in many ways, using graphs, tables, measures of center, and measures of spread.

<table>
<thead>
<tr>
<th>Graphs</th>
<th>Tables</th>
<th>Measures of Center</th>
<th>Measures of Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>bar graphs</td>
<td>frequency table</td>
<td>mean</td>
<td>range</td>
</tr>
<tr>
<td>circle graphs</td>
<td>two-way table</td>
<td>median</td>
<td>interquartile range</td>
</tr>
<tr>
<td>dot plots</td>
<td></td>
<td>mode</td>
<td>MAD</td>
</tr>
<tr>
<td>line plots</td>
<td></td>
<td></td>
<td>SD</td>
</tr>
<tr>
<td>histogram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>box plot</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scatter plot</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>line graph</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Make a table similar to the one below. Enter the types of graphs, measures of center, and measures of spread you can use with each data type.

<table>
<thead>
<tr>
<th>What can I use?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Categorical Data</strong></td>
</tr>
<tr>
<td>Graphs:</td>
</tr>
<tr>
<td>Measures of center:</td>
</tr>
<tr>
<td>Measures of spread:</td>
</tr>
</tbody>
</table>

**HINT:**
1. What kinds of graphs require finding numbers?
2. What measures of center can you observe without numbers?
3. What measures of spread can only be calculated with numbers?
Looking for Pythagoras

Investigation 1, ACE Exercise 8

The points (0, 0) and (3, 2) are two vertices of a polygon with integer coordinates.

Suppose the polygon is a square. What could the other two vertices be?

**HINT:**

1. Graph the points (0, 0) and (3, 2) on a coordinate grid.

2. Suppose the polygon is a square. What do you know about the sides and angles of the square?

3. What do you know about the slopes of two lines that are perpendicular?

4. What do you know about the slopes of two lines that are parallel?

5. Use what you know about slopes to form a square.
Looking for Pythagoras

Investigation 2, ACE Exercise 41

Find the length of every line segment that can be drawn by connecting dots on a 3 dot-by-3 dot grid.

HINT:

1. Mark off 3 dot-by-3 dot grids on a piece of dot paper.
2. Draw line segments with as many different lengths as possible by connecting dots.
3. Draw a square using each line segment as the side of the square.
4. Find the area of each square.
5. Find the length of each segment.
Looking for Pythagoras

Investigation 3, ACE Exercise 4

Use the Pythagorean Theorem to find the length of the hypotenuse of this triangle.

Hint:
1. Find the area of a square drawn with the 3-centimeter leg as one of its sides.
2. Find the area of a square drawn with the 6-centimeter leg as one of its sides.
3. What do you know about the relationship among the squares drawn on the legs and a square drawn on the hypotenuse?
4. Use this relationship to find the area of the square drawn on the hypotenuse. Then, find the length of the hypotenuse.
Looking for Pythagoras

Investigation 4, ACE Exercise 2

The Wheel of Theodorus in Problem 4.1 includes only the first 11 triangles in the wheel. The wheel can go on forever.

The Wheel of Theodorus

a. Find the side lengths of the next three triangles.
b. Find the areas of the first five triangles in the wheel. Describe any patterns you observe.
c. Suppose you continue adding triangles to the wheel. Which triangle will have a hypotenuse of length 5 units?

HINT:
1. For any triangle in the wheel, what is the length of the shortest leg? How can you use the previous triangle to help you find the length of the longer leg? Find the lengths of both legs of the twelfth triangle.
2. Use the Pythagorean Theorem to find the length of the hypotenuse of the twelfth triangle.
3. Recall the formula for the area of a triangle: \( A = \frac{1}{2}bh \). Use this formula to find the area of each triangle. What do you notice about the values of \( b \) and \( h \) for each triangle?
4. Look for a pattern among the lengths of the hypotenuses of the triangles. For which triangle will the length of the hypotenuse equal 5?
Looking for Pythagoras

Investigation 5, ACE Exercise 7

Use the figure below.

![Diagram of triangle KLM with angles and sides labeled]

a. How many 30-60-90 triangles do you see in the figure?

b. What is the perimeter of triangle $KLM$?

**HINT:**

1. Find a triangle similar to triangle $KLM$.
2. Copy the figure and label all of the angles of the three triangles.
3. Use what you know about 30-60-90 triangles to find the side lengths of the smallest triangle.
4. Which side of the small triangle corresponds to the side of triangle $KLM$ with a measure of 6 meters?
5. Now use the ratio of corresponding sides of similar triangles to find the scale factor for the small triangle and triangle $KLM$. 
Growing, Growing, Growing

Investigation 1, ACE Exercise 14

Zak's uncle wants to donate money to Zak's school. He suggests three possible. Look for a pattern in each plan.

Plan 1 He will continue the pattern in this table until day 12.

<table>
<thead>
<tr>
<th>Day</th>
<th>Donation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1</td>
</tr>
<tr>
<td>2</td>
<td>$2</td>
</tr>
<tr>
<td>3</td>
<td>$3</td>
</tr>
<tr>
<td>4</td>
<td>$4</td>
</tr>
</tbody>
</table>

Plan 2 He will continue the pattern in this table until day 10.

<table>
<thead>
<tr>
<th>Day</th>
<th>Donation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1</td>
</tr>
<tr>
<td>2</td>
<td>$3</td>
</tr>
<tr>
<td>3</td>
<td>$9</td>
</tr>
<tr>
<td>4</td>
<td>$27</td>
</tr>
</tbody>
</table>

Plan 3 He will continue the pattern in this table until day 7.

<table>
<thead>
<tr>
<th>Day</th>
<th>Donation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1</td>
</tr>
<tr>
<td>2</td>
<td>$4</td>
</tr>
<tr>
<td>3</td>
<td>$16</td>
</tr>
<tr>
<td>4</td>
<td>$64</td>
</tr>
</tbody>
</table>

a. Copy and extend each table to show how much money the school would receive each day.

b. For each plan, write an equation for the relationship between the day number \( n \) and the number of dollars donated \( d \).

c. Are any of the relationships in Plans 1, 2, or 3 exponential functions? Explain.

d. Which plan would give the school the greatest total amount of money?

HINT:

1. For each plan, find how the donations changed from day 0 to day 1 to day 2 to day 3.

2. Decide if the change is linear. That is, can you find the value for the next day by adding a fixed number or slope to the value for the day before?
Growing, Growing, Growing

Investigation 1, ACE Exercise 14 (continued)

3. Decide if the change is exponential. That is, can you find the value for the next day by multiplying the value for the previous day by a fixed number or growth factor?

4. For any of the relationships that are exponential functions, use the growth factor to write an equation for the relationship between the day number \( n \) and the number of dollars donated \( d \). Check your equation using the values given in the table.
Growing, Growing, Growing

Investigation 2, ACE Exercise 4

As a biology project, Talisha is studying the growth of a beetle population. She starts her experiment with 5 beetles. The next month she counts 15 beetles.

a. Suppose the beetle population is growing linearly. How many beetles can Talisha expect to find after 2, 3, and 4 months?

b. Suppose the beetle population is growing exponentially. How many beetles can Talisha expect to find after 2, 3, and 4 months?

c. Write an equation for the number of beetles $b$ after $m$ months if the beetle population is growing linearly. Explain what information the variables and numbers represent.

d. Write an equation for the number of beetles $b$ after $m$ months if the beetle population is growing exponentially. Explain what information the variables and numbers represent.

e. How long will it take the beetle population to reach 200 if it is growing linearly?

f. How long will it take the beetle population to reach 200 if it is growing exponentially?

HINT:

1. If a population is growing in a linear pattern, how does the number of beetles change from 5 to 15? Use this pattern of change to find the number of beetles after 2, 3, and 4 months.
Growing, Growing, Growing

Investigation 2, ACE Exercise 4 (continued)

2. If a population is growing exponentially, how does the number of beetles change from 5 to 15? Use this pattern of change to find the number of beetles after 2, 3, and 4 months.

3. In a linear equation, where is the number that represents the pattern of change?

4. In an exponential equation, where is the number that represents the pattern of change?

5. For either equation, what is the $y$-intercept?
Growing, Growing, Growing

Investigation 3, ACE Exercise 9

Maya’s grandfather opened a savings account for her when she was born. He opened the account with $100 and did not add or take out any money after that. The money in the account grows at a rate of 4% per year.

a. Make a table to show the amount in the account from the time Maya was born until she turned 10.

b. What is the growth factor for the account?

c. Write an equation for the value of the account after any number of years.

HINT:

1. Decide whether the pattern of change of Maya’s account is linear or exponential.

2. Based on your answer to Question 1, decide whether to add or to multiply by the rate of change.

3. If the value increases by 4%, what is the growth factor?

4. What is the initial value of the account?
Growing, Growing, Growing

Investigation 4, ACE Exercise 4

Penicillin decays exponentially in the human body. Suppose you receive a 300-milligram dose of penicillin to combat strep throat. About 180 milligrams will remain active in your blood after 1 day.

a. Assume the amount of penicillin active in your blood decreases exponentially. Make a table showing the amount of active penicillin in your blood for 7 days after a 300-milligram dose.

b. Write an equation for the relationship between the number of days $d$ since you took the penicillin and the amount of the medicine $m$ remaining active in your blood.

c. What is equation for a 400-milligram dose?

HINT:

1. If 180 milligrams out of the original 300 milligrams remain active in the blood after 1 day, what percent of the original penicillin remains active in the blood?

2. What is the decay factor, that is, the number you can multiply the original amount of penicillin by to find the amount of penicillin left after each day?

3. What is the original amount, or $y$-intercept, that you will use in the equation?

4. Write an exponential equation using the decay factor and $y$-intercept.

5. For part (c), what does 400 represent in the equation?
Growing, Growing, Growing

Investigation 5, ACE Exercise 63

Without actually graphing these equations, describe and compare their graphs. Be as specific as you can.

\[ y = 4^x \quad y = 0.25^x \quad y = 10(4^x) \quad y = 10(0.25^x) \]

HINT:

1. What is the initial amount, or \( y \)-intercept, given by each equation?
2. Where will you see these amounts represented on the graph?
3. Which equations have a growth factor? How do these growth factors affect the graph?
4. Which equations have a decay factor? How do these decay factors affect the graph?
Frogs, Fleas, and Painted Cubes

Investigation 1, ACE Exercise 7

The equation for the areas of rectangles with a certain fixed perimeter is \( A = \ell(20 - \ell) \), where \( \ell \) is the length in meters.

a. Describe the graph of this equation.

b. What is the maximum area for a rectangle with this perimeter? What dimensions correspond to this area? Explain.

c. A rectangle with this perimeter has a length of 15 meters. What is its area?

d. Describe two ways you can find the perimeter. What is the perimeter?

HINT:

1. Draw a rectangle and label one side \( \ell \) and the other side in terms of \( \ell \).

2. Make a table showing the length, width, and area for lengths of 0, 5, 10, 15, and 20 meters. Use this table to answer parts (a)–(c).

3. Use the information in your table to find the perimeters of the rectangles.

4. Now think about your equation and graph. If you know the length of one side of your rectangle, how can you find the length of the other side? What will you see in either the equation or the graph to help you find the perimeter of the rectangle with a side length of 15 meters?
Frogs, Fleas, and Painted Cubes

Investigation 2, ACE Exercise 36

Write each expression in factored form.

a. $x^2 + 13x + 12$ 
   b. $x^2 - 13x + 12$ 
   c. $x^2 + 8x + 12$

d. $x^2 - 8x + 12$ 
   e. $x^2 + 7x + 12$ 
   f. $x^2 - 7x + 12$

g. $x^2 + 11x - 12$ 
   h. $x^2 - 11x - 12$ 
   i. $x^2 + 4x - 12$

j. $x^2 - 4x - 12$ 
   k. $x^2 + x - 12$ 
   l. $x^2 - x - 12$

HINT:

1. Sketch a rectangle with four sections as in Problem 2.3, Question A.

2. Label the area of one section $x^2$. Label the dimensions of this square section.

3. Label the area of another section 12.

4. To find the dimensions of the section with an area of 12, think of factors of 12. Also consider that the two remaining sections must have a total area of $13x$. Therefore, the factors of 12 must also have a sum of 13. What two factors of 12 have a sum of 13?

5. Label the dimensions of the section with these factors of 12. Then use these dimensions to find the areas of the two remaining sections.
Frogs, Fleas, and Painted Cubes

Investigation 3, ACE Exercise 10

In a 100-meter race, five runners are from the United States and three runners are from Canada.

a. How many handshakes occur if the runners from one country exchange handshakes with the runners from the other country?

b. How many high fives occur if the runners from the United States exchange high fives?

HINT:

1. How many handshakes will one member of the U.S. team make with all of the members of the Canadian team?

2. How many handshakes will all five members of the U.S. team make with the three members of the Canadian team? Draw a diagram, if necessary.

3. How many high fives will one member of the U.S. team make with the other members of his team?

4. Draw a diagram to help you find the number of high fives all five members of the U.S. team will make with one another.
Frogs, Fleas, and Painted Cubes

Investigation 4, ACE Exercise 4

The highest dive in the Olympic Games is from a 10-meter platform. The height $h$ in meters of a diver $t$ seconds after leaving the platform can be estimated by the equation $h = 10 + 4.9t - 4.9t^2$.

a. Make a table of the relationship between time and height.

b. Sketch a graph of the relationship between time and height.

c. When will the diver hit the water's surface? How can you find this answer by using your graph? How can you find this answer by using your table?

d. When will the diver be 5 meters above the water?

e. When is the diver falling at the fastest rate? How is this shown in the table? How is this shown in the graph?

HINT:

1. Approximately how long do you think a diver is in the air during a dive? Use this answer to help you decide on appropriate values to use for time in your table.

2. For part (c), what is the height of a diver when the diver hits the water? Where will you see this height in your graph? Where will you see this height in your table?

3. For part (e), what do you know about the height of the diver when the diver is falling at the fastest rate? Where will you see this in your graph?

4. Look at the heights in your table and how the height changes at equal time intervals. How will you find the time when the diver is falling the fastest?
Butterflies, Pinwheels, and Wallpaper

Investigation 1, ACE Exercise 6

Quadrilateral $A'B'C'D'$ is a reflection image of quadrilateral $ABCD$.

a. On a copy of the diagram, draw the line of reflection. Explain how you found it.

b. Describe the relationship between a point on the original figure and its image point on $A'B'C'D'$.

**HINT:**

1. Draw a line segment from each vertex of quadrilateral $ABCD$ to its image.

2. What is the relationship between the line of reflection and the line segments you drew? Use your line segments to help you draw the line of reflection.

3. What do you know about the distance from a point on quadrilateral $ABCD$ to the line of reflection and the distance from a point on quadrilateral $A'B'C'D'$ to the line of reflection?
Butterflies, Pinwheels, and Wallpaper

Investigation 2, ACE Exercise 32

Alejandro wants to build a footbridge directly across a pond from point $A$ to point $B$. He needs to find the length of $AB$. He places stakes at points $A$, $B$, and $C$ to form a right triangle as shown in the diagram below.

a. Where should Alejandro place the stakes for points $D$ and $E$ to form a congruent triangle he can then measure?

b. What measurement will tell him the length of $AB$?

**HINT:**

1. What information do you need to decide whether two triangles are congruent?
2. Are the triangles congruent?
3. What side of triangle $EDC$ corresponds to side $AB$ of triangle $ABC$?
Butterflies, Pinwheels, and Wallpaper

Investigation 3, ACE Exercise 8

a. Use triangle \(ABC\) shown in the diagram.

Copy and complete the table showing the coordinates of points \(A–C\) and their images after a reflection in the line \(y = x\).

<table>
<thead>
<tr>
<th>Point</th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Coordinates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coordinates After a Reflection in (y = x)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Draw the image and label the vertices \(A', B',\) and \(C'\).

c. Add a row to your table to show the coordinates of points \(A–C\) and their images after a reflection of triangle \(A'B'C'\) in the \(x\)-axis.

d. Draw the image and label the vertices \(A'', B'',\) and \(C''\).

e. Draw the image of triangle \(ABC\) after the same two reflections, but in the reverse order. That is, reflect triangle \(ABC\) in the \(x\)-axis and then reflect its image, triangle \(A'B'C'\), in the line \(y = x\). What does the result suggest about the commutativity of a sequence of line reflections?
Butterflies, Pinwheels, and Wallpaper

Investigation 3, ACE Exercise 8 (continued)

HINT:

1. What combination of coordinate rules reflects points over the line $y = x$ and then reflects the points over the $x$-axis?

2. What combination of coordinate rules reflects points over the $x$-axis and then reflects the points over the line $y = x$?

3. Are the results the same?
Butterflies, Pinwheels, and Wallpaper

Investigation 4, ACE Exercises 7–10

For Exercises 7–10, suppose you dilate quadrilateral $ABCD$ by a scale factor of 2. Then you flip, turn, or slide the image to quadrilateral $A''B''C''D''$.

7. Describe how the side lengths of $A''B''C''D''$ are related to the side lengths of each quadrilateral.
   a. $ABCD$
   b. $A'B'C'D'$

8. Describe how the perimeter of quadrilateral $A''B''C''D''$ is related to the perimeter of each quadrilateral.
   a. $ABCD$
   b. $A'B'C'D'$

9. Describe how the area of quadrilateral $A''B''C''D''$ is related to the area of each quadrilateral
   a. $ABCD$
   b. $A'B'C'D'$

10. Describe how the slopes of the sides of quadrilateral $A''B''C''D''$ are related to the slopes of the sides of each quadrilateral.
    a. $ABCD$
    b. $A'B'C'D'$

HINT:

1. What properties are preserved after a dilation?
2. What properties are *not* preserved after a dilation?
3. What properties are preserved after a flip, turn, or slide?
4. What properties are *not* preserved after a flip, turn, or slide?
Say It With Symbols

Investigation 1, ACE Exercise 3

A rectangular pool is $L$ feet long and $W$ feet wide. A tiler makes a border by placing 1-foot-square tiles along the edges of the pool and triangular tiles on the corners, as shown. The tiler makes the triangular tiles by cutting the square tiles in half along a diagonal.

a. Suppose the pool is 30 feet long and 20 feet wide. How many square tiles does the tiler need for the border?

b. Write two equations for the number of square tiles $N$ needed to make this border for a pool $L$ feet long and $W$ feet wide.

c. Explain why your two expressions are equivalent.

**HINT:**

1. How many sides of the pool have a measure of 30 feet? How many sides have a measure of 20 feet? How many tiles are needed for these sides?

2. How many tiles are needed to make the corners?

3. Find the total number of tiles needed to surround the pool.

4. Replace the width of 20 with the variable $W$ and the length of 30 with the variable $L$ to write an equation to represent the total number of tiles $N$.

5. Think of another way to represent the total number of tiles needed to make the border of the pool.
Say It With Symbols

Investigation 2, ACE Exercise 8

Recall the equations form Problem 2.2, \( P = 2.50V - 500 \) and \( V = 600 - 500R \).

The manager estimates the daily employee-bonus fund \( B \) (in dollars) from the number of visitors \( V \) using the equation \( B = 100 + 0.50V \).

a. Suppose the probability of rain is 30%. What is the daily employee-bonus fund?

b. Write an equation that relates the employee-bonus fund \( B \) to the probability of rain \( R \).

c. Suppose the probability of rain is 50%. Use your equation to calculate the employee-bonus fund.

d. Suppose the daily employee-bonus fund is $375. What is the probability of rain?

HINT:

1. Suppose the probability of rain is 30%. What is the number of visitors expected at the park?

2. Use the number of visitors expected at the park to determine the amount of the employee bonus in dollars.

3. Use the bonus equation \( B = 100 + 0.50V \). Substitute an equivalent expression for the number of visitors into the equation in place of \( V \).

4. Use this new equation for parts (c) and (d).
Say It With Symbols

Investigation 3, ACE Exercise 3

According to the equation \( V = 200 + 50(T - 70) \), the number of visitors \( V \) to a park depends on the day's high temperature \( T \) (in degrees Fahrenheit). Suppose 1,000 people visited the park one day. Predict that day's high temperature.

**HINT:**

1. For which variable can you substitute the number of visitors, 1,000 people, into the equation to find that day's high temperature?

2. Simplify the equation using the Distributive and Commutative properties.

3. Solve the resulting equation using the appropriate properties of equality.

4. After you solve the equation algebraically, you can use a table or graph of the equation to check your work.
Say It With Symbols

Investigation 4, ACE Exercise 4

A middle school orders some yearbooks. Their bill is shown below.

The school gives some free copies to the yearbook advisor and staff. They sell the rest to students. The equation below tells how close the school is to paying for the printing bill.

\[ y = 2,500 - 15(N - 8) \]

Describe what information the numbers and variables represent in this situation.

**HINT:**

1. What does the number 2,500 represent in this situation?
2. What do you think the variable \( N \) represents in this situation?
3. Since 8 is subtracted from \( N \), what do you think 8 represents in this situation?
4. Since \( (N - 8) \) is multiplied by 15, what do you think the number 15 represents in this situation?
5. Since the equation represents how close the school is to paying the bill, what does \( y \) represent in this situation?
**Say It With Symbols**

**Investigation 5, ACE Exercise 7**

Look at the product of three consecutive whole numbers. For example:

\[
1 \times 2 \times 3 \quad 2 \times 3 \times 4 \quad 3 \times 4 \times 5
\]

a. What pattern do you see?

b. Make a conjecture about the product of three consecutive whole numbers. Show that your conjecture is true.

**HINT:**

1. Find the products of each of the three sets of consecutive numbers.

2. What do you notice about the products?

3. Suppose you have three consecutive integers. What are the possibilities for odd and even combinations?

4. Use what you know about the product of odd and even numbers to make your conjecture.
It’s In the System

Investigation 1, ACE Exercise 2

Kateri saves her quarters and dimes. She plans to exchange the coins for paper money when the total value equals $10.

a. How many coins does she need to make $10 if all the coins are quarters? If all the coins are dimes?

b. What equation relates the number of quarters $x$ and the number of dimes $y$ to the goal of $10$?

c. Use the answers from part (a) to help you draw a graph showing all solutions to the equation.

d. Use the graph to find five combinations of dimes and quarters that will allow Kateri to reach her goal.

HINT:

1. What is the value of a quarter? What is the value of a dime? Use this information in the Exercise.

2. Use your answers from Question 1 to write an equation relating the number of quarters $x$, the number of dimes $y$, and the total value of $10$.

3. Plot the two solutions from part (a) on your graph.

4. Look for a pattern among these points that will help you find five combinations of dimes and quarters that will allow Kateri to reach her goal.
It’s In the System

Investigation 2, ACE Exercise 1

A school is planning a Saturday Back-to-School Festival to raise funds for the school art and music programs. Some of the planned activities are a ring toss, frog jump, basketball free throws, and a golf putting green. The organizers are considering two pricing plans.

Plan 1: $5 admission fee, $1 per game
Plan 2: $2.50 admission fee, $1.50 per game

a. Write equations that show how the cost \( y \) for playing the games at the festival depends on the number of games \( x \) that a participant chooses to play.

b. Estimate the coordinates of the intersection point of the graphs of the two equations. Check to see if those coordinates are an exact solution of both equations.

c. Use the expressions in the two cost equations to write and solve a single linear equation for the \( x \)-coordinate of the intersection point. Then use that \( x \)-value to find the \( y \)-coordinate of the intersection point.

d. For what number of games would Plan 1 be a better deal for participants than Plan 2?

HINT:

1. For each plan, what is the fixed cost? How is the fixed cost represented in the equation?

2. For each plan, what is the cost per game? How is the cost per game represented in the equation?

3. What do the coordinates of the point of intersection represent in this situation? Use this information to help you estimate the coordinates.

4. Making a graph of the two equations may help you answer part (d).
It’s In the System

Investigation 3, ACE Exercise 2

Mariana lives 1,250 meters from school. Ming lives 800 meters from school. Both students leave for school at the same time. Mariana walks at an average speed of 70 meters per minute, while Ming walks at an average speed of 40 meters per minute. Mariana’s route takes her past Ming’s house.

a. Write equations that show Mariana and Ming’s distances from school \( t \) minutes after they leave their homes.

Answer parts (b)–(d) by writing and solving equations or inequalities.

b. When, if ever, will Mariana catch up with Ming?

c. How long will Mariana remain behind Ming?

d. At what times is the distance between the two students less than 20 meters?

HINT:

1. Sketch a diagram of the school, Ming's house, and Mariana’s house. Label the known distances.

2. What is the relationship among the distance traveled, the rate of speed, and the time it takes to walk at the given speed? Use this relationship to write equations for Mariana and for Ming given their distance traveled and speed.

3. What do you know about the students’ distance from school if Mariana has caught up with Ming? Use this information to write an equation and then solve it for time.

4. What do you know about Mariana’s distance if she is behind Ming? Use this fact to write an inequality and then solve it for time.

5. Write an inequality to find when the students are 20 meters from each other. How can you show that the difference of the distances will each be less than or equal to 20 meters? Solve the inequality for time.
It’s In the System

Investigation 4, ACE Exercise 9

Math Club members want to advertise their fundraiser each week in the school paper. They know that a front-page ad is more effective than an ad inside the paper. They have a $30 advertising budget. It costs $2 for each front-page ad and $1 for each inside-page ad. The club wants to advertise at least 20 times.

a. What are some possibilities for the numbers of front-page ads and the number of inside-page ads the club can place?

b. Write a system of linear inequalities to model this situation.

c. Graph your system of inequalities. Be sure it is clear which region shows the solution.

HINT:

1. Suppose the Math Club placed 1 front-page ad. How much money is left for inside-page ads? How many inside-page ads is this? What is the total cost of the ads?

2. Suppose the Math Club placed 10 front-page ads. How much money is left for inside-page ads? How many inside-page ads is this? What is the total cost of the ads?

3. Can the Math Club place more than 10 front-page ads? Why?

4. Suppose you let \( x \) represent the number of front-page ads and \( y \) represent the number of inside-page ads. Write an inequality to model the total number of times the club wants to advertise.

5. Suppose you let \( x \) represent the number of front-page ads and \( y \) represent the number of inside-page ads. Write an inequality to model the total amount of money that the club can use for advertising.
Function Junction

Investigation 1, ACE Exercise 2

The graphs below show the pattern of time and distance traveled by two school buses. Make a copy of each graph. On copies of each graph mark the following intervals:

- when the bus is speeding up
- when the bus is slowing down
- when the bus is moving at a constant speed
- when the bus is stopped

a. Bus A  

b. Bus B

HINT:

1. When a bus is speeding up, how does its distance change? How is this represented in the graph?
2. When a bus is slowing down, how does its distance change? How is this represented in the graph?
3. What does a graph that represents a constant rate look like?
4. When a bus is stopped, does its distance change? How is this represented in the graph?
Function Junction

Investigation 2, ACE Exercises 7–16

For Exercises 7–16, study each number pattern to see if it begins an arithmetic sequence, a geometric sequence, or neither. For those that begin either an arithmetic sequence or a geometric sequence, do the following.

1. Tell which type of sequence, arithmetic or geometric, is shown
2. Write an equation relating \( s(n) \) and \( s(n + 1) \)
3. Write an algebraic expression for a function \( s(n) \) that shows how to find any term in the sequence beyond those already given

7. \(-5, 1, 7, 13, 19, 25, 31, \ldots\)
8. \(16, 13, 10, 7, 4, 1, -2, \ldots\)
9. \(10, 8, 6, 4, 2, 0, 0, 0, \ldots\)
10. \(5, -10, 20, -40, 80, -160, \ldots\)
11. \(3, 4.5, 6, 7.5, 9, 10.5, 12, \ldots\)
12. \(3, 2, 1, 0, 1, 2, 3, 2, 1, \ldots\)
13. \(27, 18, 12, 8, \frac{16}{3}, \frac{32}{9}, \frac{64}{27}, \ldots\)
14. \(4, 4, 4, 4, 4, \ldots\)
15. \(1, -1, 1, -1, 1, -1, 1, \ldots\)
16. \(1, 5, 25, 125, 625, 3125, \ldots\)

HINT:

1. Is there a common ratio between each number in the sequence?
2. Is there a common difference between each number in the sequence?
3. How is the common ratio or common difference represented in the algebraic expression?
Function Junction

Investigation 3, ACE Exercises 11–15

Exercises 11–15 refer to the following figure.

For each flag:
- Give the coordinate rule \((x, y) \rightarrow (\text{[flag]}, \text{[flag]})\) for the transformation that maps the red flag to the given flag.
- Identify the kind of transformation(s) involved.

11. Flag A
12. Flag B
13. Flag C
14. Flag D
15. Flag E

**HINT:**
1. Write the coordinates of the four key points on each flag.
2. Match each key point on Flag A with its corresponding key point on the red flag. Look for a relationship between the coordinates of each pair of key points. Repeat this process for the other flags.
Function Junction

Investigation 3, ACE Exercises 11–15 (continued)

3. What kinds of transformations produce an image that is the same size as the original figure? What kinds of transformations produce an image that is larger or smaller than the original figure?

4. What kinds of transformations produce an image that faces the same direction as the original figure? What kinds of transformations produce an image that faces a different direction than the original figure?
Function Junction

Investigation 4, ACE Exercises 9–14

For Exercises 9–14, each quadratic function is in standard form.

• Complete the square to write each function in vertex form.
• Identify coordinates of the maximum or minimum point.
• Identify the x-intercept(s) and y-intercept.
• State which form is more convenient to identify coordinates of the maximum/minimum point, x-intercept(s) and y-intercept.

9. \( f(x) = x^2 + 2x - 3 \)  
10. \( g(x) = x^2 - 4x - 5 \)
11. \( h(x) = x^2 - 6x + 5 \)  
12. \( j(x) = x^2 + 4x + 2 \)
13. \( k(x) = -x^2 + 3x - 1 \)  
14. \( l(x) = -x^2 + 8x - 5 \)

HINT:

1. For each function, what number do you need to add and subtract from the right side of the equation?
2. How are the coordinates of the maximum or minimum point represented in vertex form of a quadratic expression?
3. How can you find the x-intercept(s) and y-intercept of a quadratic function written in standard form?
4. How can you find the x-intercept(s) and y-intercept of a quadratic function written in vertex form?
5. Which form do you prefer for identifying coordinates of the minimum/maximum point, x-intercept(s), and y-intercept? Explain.
Function Junction

Investigation 5, ACE Exercises 6–10

Write the sums and differences as equivalent standard polynomial expressions.

6. \((x^2 - x - 2) + (3x^2 + 5x + 10)\)
7. \((3x^2 - 4x + 7) - (5x^2 - x)\)
8. \((5x^3 + 3x^2 + 4x - 3) + (x^3 - 7x^2 - 4x + 1)\)
9. \((x^3 - 7x^2 - 4x + 1) - (5x^3 + 3x^2 + 4x - 3)\)
10. \((3x^3 + 5x^2 + 10) + (7x^4 + 5x^3 - 10x^2 + 4x)\)

HINT:

1. In a difference of polynomials, what effect does the subtraction sign have on the terms of the expression being subtracted?

2. When adding or subtracting polynomials, how can you determine which terms can be combined?

3. When writing a polynomial in standard form, be sure to list the terms in order from greatest exponent to least exponent.