Learning Objectives

After studying this chapter, you will be able to:

• Evaluate risk in solutions to optimization models using Monte Carlo simulation.
• Solve optimization models with chance constraints.
• Use multiple parameterized simulations in Analytic Solver Platform to find optimal solutions in simulation models with decision variables.

• Use Analytic Solver Platform to combine simulation modeling and optimization to maximize or minimize the expected value of a model output.
• Incorporate uncertainty into optimization models such as project selection.
In the previous chapters on linear, integer, and nonlinear optimization, we have assumed that all data in the models are deterministic—that is, known with certainty. It is rare that any optimization model is completely deterministic; in most cases, some of the data will be uncertain. This implies that inherent risk exists in using the optimal solution obtained from a deterministic model. As we noted in Chapter 1, such models are called stochastic.

This chapter provides an introduction to stochastic optimization models. We discuss how to incorporate probability concepts, Monte Carlo simulation methods, and other analytical approaches to deal with stochastic models. We restrict our discussions and examples to relatively simple situations, although we do note that many other advanced techniques are available to handle specialized situations.

**Risk Analysis in Optimization**

If an optimization model has uncertain variables, we might first solve it deterministically and then use Monte Carlo simulation to analyze the results.

**EXAMPLE B.1 Uncertainty in the Sklenka Ski Model**

In the Sklenka Ski Company (SSC) model developed in Chapter 13, suppose that the labor intensity associated with finishing a pair of skis is uncertain. If more than 21 hours are required (recall that this is a binding constraint), then overtime will be needed. Assume that the finishing time for a pair of Jordanelle skis is triangular with a minimum value of 0.9, a most likely value of 1.0, and a maximum value of 1.3 and that finishing time for a pair of Deercrest skis is also triangular with a minimum of 1.4, a most likely value of 1.5, and a maximum value of 1.8. How often will overtime be needed if 5.25 Jordanelle and 10.5 Deercrest skis, the optimal solution, are scheduled each day?

In the *Sklenka Skis* spreadsheet (see Figure B.1), define the finishing parameters in cells B7 and C7 as uncertain variables using Analytic Solver Platform with the specified triangular distributions and the hours used for finishing (cell D16) as an uncertain output cell. Figure B.2 shows the frequency distribution of the total number of hours required in the finishing department after simulating the model. By setting the Lower Cutoff in the Chart Statistics field as 21 hours, we see that the likelihood that overtime will be needed is about 85%.

**Chance Constraints**

In the previous example, requiring overtime too often may not be desirable. Suppose that the company wants to determine a daily schedule so that the probability of overtime—that is, requiring more than 21 hours of finishing time—is less than 0.1, or 10% of the time. This can be done using a *chance constraint*. A chance constraint is one that specifies the fraction of trials in a simulation that must satisfy a constraint. In this case, we would want to specify that the percentage of trials requiring less than 21 hours of finishing time is at least 80%.
Chance constraints are defined by a percentile, or value at risk (VaR), measure. *Solver* has two types of value at risk measures. A VaR constraint with chance $p\%$ requires that the constraint be satisfied $p\%$ of the time. This does not consider the magnitude of the violation when the constraint is not satisfied. An alternative measure is called conditional value at risk (CVaR). A conditional value at risk constraint places a bound on the average magnitude of all violations of the constraint that may occur $(1 - p)\%$ of the time and is more conservative in nature. Generally, CVaR is better to use for very large problems because of its mathematical properties; however, we illustrate only the VaR constraint.
EXAMPLE B.2 Solving the SSC Model with a Chance Constraint

To include a chance constraint in the SSC Solver model, click on Chance in the Solver Parameters dialog and then the Add button. In the Add Constraint dialog, specify the left and right-hand sides of the constraint as you would usually do; however, in the drop-down box on the right, choose VaR and specify the probability of meeting the constraint below it, as shown in Figure B.3. You must also uncheck the original constraint for finishing hours (or delete it from the Solver model) in the Normal Constraints section (as shown in Figure B.4). The solution is shown in Figure B.5. In comparison with the original optimal solution, we are producing a smaller amount of each type of ski to reduce the total number of finishing hours required. A simulation of the model (see Figure B.6) shows that the chance constraint is not only satisfied, but oversatisfied, with only a small chance of overtime.

Solver typically finds a conservative solution to problems with chance constraints. However, Analytic Solver Platform can automatically improve the solution by adjusting the size of the uncertainty set for the chance constraint auto-adjust process. To do this, click the button that looks like a little green arrow inside a circle (not the “play” button on the right!) at the top of the task pane in the Output tab (see Figure B.5). Figure B.7 shows the results. As you can see, the maximum profit increases, and the chance of exceeding the finishing hours constraint is just a bit less than 20%, much closer to the specified requirement.
Supplementary Chapter B  Optimization Models with Uncertainty

Figure B.5
Solution to the SSC Model with a Chance Constraint

Figure B.6
Simulation Results for the SSC Model with Chance Constraint
Lockheed Martin Space Systems Company, based in Denver, Colorado, is located on a 5,500-acre facility with 37 major buildings—most constructed prior to 1970. Each major building contains a multitude of systems, all exhibiting various degrees of wear and tear. The Facility Operations and Services (FO&S) Department at Lockheed Martin is responsible for choosing a portfolio of projects to improve the condition of the site. Allocating capital to the maintenance and modernization of the facility infrastructure and office space is a nontrivial task, given the size of the facility. Previously, FO&S selected projects based on the judgment of decision makers within the department and qualitative input from various stakeholders. This lengthy process often involved stressful negotiations but produced suboptimal decisions.

Lockheed Martin needed a robust analytical method for optimizing its project selection. They developed a spreadsheet-based decision support system that used chance-constrained programming to address the risks and constraints inherent in capital-rationing problems. The team used financial data on projects implemented during the previous 5 years and followed an analytical approach to update management’s estimates on the mean and variance of each project’s spending. They then constructed a chance constraint to ensure that the choice of portfolios stays within the budget plus a small tolerance for overrun with at least a predetermined probability.

The team used Microsoft Excel to develop the model, solve the optimization problem, and present the results. The optimization was implemented using Premium Solver and incorporated 267 binary decision variables and two constraints. The numerical results showed that the optimization tool improved the project-selection process in three areas:

- better use of available budget
- reduction in carryover risk (i.e., money spent to fund a project that took longer than expected);
- increase in overall utility to the department

While implementing the model at Lockheed Martin and during subsequent interactions with decision makers, the team also highlighted how the tool could be used to update management’s estimates on the mean and variance of each project’s spending.

makers and other users, the team observed that using spreadsheets to construct, solve, and present the analytical model was one of the reasons management received it well. In addition, their analytic approach changed the way that decision makers handled the capital allocation problem. The quantitative approach reduced time and effort spent on discussions and negotiations; elevated the importance of keeping historical data; and clearly specified projects that management had previously excluded or mandated in an objective setting.

**Service Levels in the Economic Order Quantity Model**

In Supplementary Chapter A, we developed a model for finding the economic order quantity to manage inventory purchasing. In most practical situations, it is unreasonable to assume that the demand is constant over time; instead, the demand in a given period of time would be random and described by a probability distribution. Recall that we stated that the reorder point is calculated as the demand during the lead time, or \( r = D t \), where \( D \) is the annual demand and \( t \) is the lead time, the time from the placement of the order until it is received. If \( D \) is uncertain, then the demand during the lead time will also be uncertain. This impacts how the reorder point should be chosen. We can use Monte Carlo simulation to analyze the optimal solution.

**EXAMPLE B.3 Finding the Distribution of Lead-Time Demand**

We will use the same model as in Example A.5 but assume that the distribution of demand is normal with a mean of 15,000 and standard deviation 2,000. Figure B.8 shows a modified spreadsheet that incorporates this assumption. Cell B5 is defined to be normally distributed using the function \( = \text{PsiNormal}(15000, 2000) \). In cell B22, we calculate the lead-time demand by multiplying the annual demand rate (B13) by the lead time in B21 and define it as an output cell. Figure B.9 shows the distribution of the lead-time demand. This chart shows that using a reorder point of 288, we will run out of stock half the time and incur quite a few dissatisfied customers.
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One way to resolve this is to set a service level. This is a constraint that represents the probability that demand can be satisfied. For example, we might want to ensure that demand can be satisfied 95% of the time. We can identify the reorder point for a particular service level from the frequency chart.

**EXAMPLE B.4  Finding the Reorder Point for a Service Level Requirement**

In the distribution of lead-time demand shown in Figure B.9, first set the Lower Cutoff value in the Chart Statistics pane to zero and then set the Likelihood value to 0.95. This will calculate the Upper Cutoff value so that the cumulative probability is 0.95. Figure B.10, for example, shows that a 95% service level requires a reorder point of about 351 units. The additional $351 - 288 = 63$ units above the mean lead-time demand is called safety stock, and it increases the total cost because these units must be held in inventory. The cost increase is calculated by multiplying the holding cost by the safety stock, or $0.2(22.00)(63) = 277.20$. 

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**Figure B.9**

Distribution of Lead-Time Demand

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**Figure B.10**

Finding the Reorder Point for a Service Level of 95%
The service level concept can be applied to other types of problems. In the next example, we analyze the hotel pricing problem that we introduced in Supplementary Chapter A when uncertainty exists in some of the model parameters.

**Hotel Pricing Model with Uncertainty**

Refer back to the Hotel Pricing Model in Supplementary Chapter A; the model is shown again in Figure B.11. In this problem, the price-demand elasticities of demand are only estimates and most likely are quite uncertain. Because we probably will not know anything about their distributions, let us conservatively assume that the true values might vary from the estimates by plus or minus 25%. Thus, we can model the elasticities by uniform distributions. Using the optimal prices identified by Solver, let us see what happens to the prediction of the number of rooms sold under this assumption using Monte Carlo simulation.

**EXAMPLE B.5 Simulating the Optimal Solution to the Hotel Pricing Model**

In the spreadsheet model, select cells D7:D9 as uncertain variable cells and define uniform distributions using Analytic Solver Platform having minimum and maximum values equal to 75% and 125% of the estimated values, respectively. These result in the following parameters for the uniform distribution:

<table>
<thead>
<tr>
<th>Cell</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>D7</td>
<td>−1.875</td>
<td>−1.125</td>
</tr>
<tr>
<td>D8</td>
<td>−2.5</td>
<td>−1.5</td>
</tr>
<tr>
<td>D9</td>
<td>−1.25</td>
<td>−0.75</td>
</tr>
</tbody>
</table>

The projected total number of rooms sold (E18) is defined as the output cell. The model was simulated for 10,000 trials, creating the report in Figure B.12. We see that the mean number of rooms sold under these prices is 450, which should be expected, since the mean values of the elasticities were used to derive the optimal prices. However, because of the uncertainty associated with the elasticities, the probability that more than 450 rooms will be sold (demanded) is about 0.5. This suggests that if the assumptions of the uncertain elasticities are true, the hotel might anticipate that demand will exceed its room capacity about half the time, resulting in many unhappy customers.

We could experiment with the frequency chart, however, to identify the appropriate hotel capacity to ensure, for example, only a 10% chance exists that demand will exceed capacity given the uncertainties we defined in the model.
EXAMPLE B.6 Identifying Hotel Capacity to Meet Service Level Constraint

By changing the Upper Cutoff value in the task pane, we could identify the likelihood of exceeding that value. Figure B.13 shows the frequency chart when the Upper Cutoff value is set to 457. We see that the likelihood of exceeding 457 rooms is fairly close to 10%. So, if we shift the capacity constraint down by 7 rooms to 443 and find the optimal prices associated with this constraint, we would expect demand to exceed 450 at most 10% of the time. Solving the original model (with the constant elasticities) and a room capacity of 443 results in the optimal prices shown in Figure B.14. Figure B.15 shows the results of a simulation using these prices, confirming that demand will exceed 450 less than 10% of the time using these new prices. Therefore, the hotel should never book more than 443 rooms.
These examples illustrate how Monte Carlo simulation can easily help to assess risks associated with solutions to optimization problems when uncertainty exists in the data. The ability of Analytic Solver Platform to combine simulation and optimization within an Excel environment provides powerful and unique analytic capabilities. In the next section, we discuss how to use Analytic Solver Platform to optimize Monte Carlo simulation models.

**Optimizing Monte Carlo Simulation Models**

In Chapter 12, we developed Monte Carlo simulation models that involved decision variables; for example, in the newsvendor model we could control the purchase quantity, and in the hotel overbooking model, we could control the number of reservations to accept. In both these models, the objective is to find the best value of the decision variables to maximize either the expected profit or net revenue. Although Analytic Solver Platform makes it easy to conduct what-if analyses by changing the decision variables and quickly running new simulations, doing so can be tedious. Fortunately, Analytic Solver Platform provides the capability, called multiple parameterized simulations, to automatically run simulations for a range of values and identify the best value for the decision variables.
Supplementary Chapter B  Optimization Models with Uncertainty

Optimizing the Newsvendor Model Using Multiple Parameterized Simulations

We use multiple parameterized simulations to vary a parameter, the purchase quantity in the newsvendor model, perform a simulation over a range of values, and summarize the results.

EXAMPLE B.7 Using Multiple Parameterized Simulations

We will use the Newsvendor Model with Historical Data spreadsheet and resampling (see Example 12.14). First, set the demand in cell B11 to =PsiDisUniform(D2:D21). Then select cell B12, the purchase quantity. From the Analytic Solver Platform ribbon, click the Parameters button and choose Simulation. In the Function Arguments dialog, set a lower limit of 40 and an upper limit of 51 (the range of the historical data). You need not enter a base case value. We will run 12 simulations, one for each value of the purchase quantity from 40 to 51. To change the number of simulations to run, click the Options button and set Simulations to Run to 12 in the Simulation tab. Next, select cell B17 (profit), click on the Results button, choose Mean from the Statistics category, and select a range of 12 cells (we chose G2 to G13). Analytic Solver Platform will place the mean for the first simulation corresponding to a purchase quantity of 40 in the first cell of this range, the mean profit for the second simulation corresponding to a purchase quantity of 41 in the second cell of this range, and so on. Finally, choose Run Once from the Simulate button options. Figure B.16 shows the results after we have customized the spreadsheet by adding the purchase quantities in column F that correspond to the means in column G to make the results more understandable. We see that the best purchase quantity to maximize the expected profit is 45.

Optimizing the Hotel Overbooking Model Using Multiple Parameterized Simulations

We may use multiple parameterized simulations in a similar fashion to find the best number of reservations to accept in the hotel overbooking model.
EXAMPLE B.8  Optimizing the Hotel Overbooking Model

In the Hotel Overbooking Monte Carlo Simulation Model with Custom Demand spreadsheet, select cell B13, the number of reservations made, and define lower and upper simulation parameters in a similar fashion as in the news-vendor example. We set the lower limit to 300 and the upper limit to 330. Therefore, we will run 31 simulations; that number needs to be set in the Simulation tab from the Options button. Next, select cells B17 (overbooked customers) and cell B18 (net revenue) individually, click on the Results button, choose Mean from the Statistics category, and select the output range for the means of each simulation. Finally, choose Run Once from the Simulate button options. Figure B.17 shows the results. We see that the best number of reservations to accept is 313, with an average of 1.64 overbooked passengers and an average net revenue of just under $36,000. As the number of reservations made increases, net revenue begins to drop and the number overbooked rises rapidly.

Simulation Optimization Using Analytic Solver Platform

Running multiple parameterized simulations is one approach for identifying optimal solutions to simulation models; however, Analytic Solver Platform can do this directly and more efficiently. We illustrate this using the hotel overbooking model.
Optimization Models with Uncertainty

A Portfolio Allocation Model

An investor has $100,000 to invest in four assets. The expected annual returns and minimum and maximum amounts with which the investor will be comfortable allocating to each investment follow:

<table>
<thead>
<tr>
<th>Investment</th>
<th>Annual Return</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Life insurance</td>
<td>5%</td>
<td>$2,500</td>
<td>$5,000</td>
</tr>
<tr>
<td>2. Bond mutual funds</td>
<td>7%</td>
<td>$30,000</td>
<td>None</td>
</tr>
<tr>
<td>3. Stock mutual funds</td>
<td>11%</td>
<td>$15,000</td>
<td>None</td>
</tr>
<tr>
<td>4. Savings account</td>
<td>4%</td>
<td>None</td>
<td>None</td>
</tr>
</tbody>
</table>

The major source of uncertainty in this problem is the annual return of each asset. In addition, the decision maker faces other risks—for example, unanticipated changes in inflation or industrial production, the spread between high- and low-grade bonds, and the spread between long- and short-term interest rates. One approach to incorporating such risk factors in a decision model is arbitrate pricing theory (APT). APT provides estimates of the sensitivity of a particular asset to these types of risk factors. Let us assume that the risk factors per dollar allocated to each asset have been determined as follows:

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Supplementary Chapter B  Optimization Models with Uncertainty

<table>
<thead>
<tr>
<th>Investment</th>
<th>Risk Factor/Dollar Invested</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Life insurance</td>
<td>-0.5</td>
</tr>
<tr>
<td>2. Bond mutual funds</td>
<td>1.8</td>
</tr>
<tr>
<td>3. Stock mutual funds</td>
<td>2.1</td>
</tr>
<tr>
<td>4. Savings account</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

The investor may specify a target level for the weighted risk factor, leading to a constraint that limits the risk to the desired level. For example, suppose that our investor will tolerate a weighted risk per dollar invested of at most 1.0. Thus, the weighted risk for a $100,000 total investment will be limited to 100,000. If our investor allocates $5,000 in life insurance, $50,000 in bond mutual funds, $15,000 in stock mutual funds, and $30,000 in a savings account (which fall within the minimum and maximum amounts specified), the total expected annual return would be

\[
0.05(5,000) + 0.07(50,000) + 0.11(15,000) + 0.04(30,000) = 6,600
\]

However, the total weighted risk associated with this solution is

\[
-0.5(5,000) + 1.8(50,000) + 2.1(15,000) - 0.3(30,000) = 110,000
\]

Because this is greater than the limit of 100,000, this solution could not be chosen.

The decision problem, then, is to determine how much to invest in each asset to maximize the total expected annual return, remain within the minimum and maximum limits for each investment, and meet the limitation on the weighted risk.

**EXAMPLE B.10  Setting Up the Spreadsheet Model**

A spreadsheet for this problem is shown in Figure B.19 (Excel file Portfolio Allocation Model). Problem data are specified in rows 4 through 10. On the bottom half of the spreadsheet, we specify the decision variables (amounts invested) in cells B16:B19, objective function (total expected return) in cell D20, and constraint functions (the total weighted risk and total amount invested) in cells D16 and B20. You can see that this particular solution is not feasible because the total weighted risk exceeds the limit of 100,000.

We assume that the annual returns for life insurance and mutual funds are uncertain but that the rate for the savings account is constant. We make the following assumptions about these uncertain variables:

- life insurance annual return, cell B6: uniform distribution with minimum 4% and maximum 6%
- bond mutual funds annual return, cell B7: normal distribution with mean 7% and standard deviation 1%
- stock mutual funds annual return, cell B8: lognormal distribution with mean 11% and standard deviation 4%
EXAMPLE B.11  Setting Up the Simulation Model

Using either the Psi functions or the distributions button in the Analytic Solver Platform ribbon to enter the parameters for the uncertain variables, the formulas in cells B6:B8 should be

- cell B6: =PsiUniform(4%, 6%)
- cell B7: =PsiNormal(7%, 1%)
- cell B8: =PsiLogNormal(11%, 4%)

The output cell is the total expected return, cell D20; define this by clicking Results/Output/In Cell from Simulation Model group in the ribbon. At this point, we may simulate any set of investment amounts and conduct what-if analyses.

Next, set up the optimization model. We need to define the decision variables, objective, and constraints. In this example, we have two constraints as well as bounds on the variables. The first constraint limits the total weighted risk to 100,000, and the second ensures that we do not allocate more than $100,000 in total to all assets. In the context of the spreadsheet, the risk constraint is expressed as D16 ≤ E10, and the allocation constraint is B20 ≤ B10. The bounds are specified by the constraints B16 ≥ C6, B16 ≤ D6, B17 ≥ C7, and B18 ≥ C8.
EXAMPLE B.12  Setting Up the Optimization Model

Select the range B16:B19, click on the Decisions button in the Optimization Model group in the Analytic Solver Platform ribbon, and select Normal. Select the constraint function cell D16, and click on the Constraints button; choose Normal and then \( \leq \). Add the right-hand side of the constraint to the Add Constraint dialog by referencing cell E10 as you would normally do in any optimization problem. Select cell B20 and repeat this procedure for the allocation constraint \( B20 \leq B10 \). Next, add the bounds specified by the minimum and maximum values in columns C and D of the spreadsheet by selecting a variable cell, clicking on Constraints, choosing Variable Type/Bound, and selecting the appropriate inequality and referencing the cell corresponding to the constraint’s right-hand side. Finally, select the objective cell D20, and click on the Objective button. Select Max category and choose Expected from the list. Click the Optimize button in the Solve Action group in the ribbon, and Analytic Solver Platform will find the optimal solution.

You can view both the optimization and simulation models by clicking the Model button on the Analytic Solver Platform ribbon. This displays the “task pane” on the right side of the spreadsheet. You may also use the task pane to create your model. Figure B.20 shows the solution found, along with the task pane that shows the model elements.

Note, however, that the value of the total expected return shown in the spreadsheet results is simply one sample from a simulation using the optimal allocations. Figure B.21 shows the distribution of the expected return; the mean (which was maximized through the optimization) is approximately $6,900, but you can see that the actual return can be lower or higher, with a small chance of exceeding $10,000.

Project Selection

Project selection and capital-budgeting models, which we studied in Chapter 15, often have many uncertainties that managers must deal with. For example, the returns and resource requirements, which are usually just estimated, are typically uncertain. In addition,
implementing a project does not guarantee that it would be successfully completed; we might be able to estimate the probability that a project will be successful. Again, using Analytic Solver Platform, these uncertainties can be easily incorporated into an optimization model.

**EXAMPLE B.13  A Project-Selection Model with Uncertainty**

Refer back to Example 15.5 (Hahn Engineering Project Selection). Suppose that the expected returns are uncertain and can be modeled using lognormal distributions. Also, assume that some projects are riskier than others and have different probabilities of being completed successfully. These assumptions are incorporated into the data shown in Table B.1. Figure B.22 shows the modified spreadsheet.

In the Model section of the spreadsheet, we need to incorporate the uncertain assumptions. We can model whether a project is successful or not using the binomial distribution with \( n = 1 \) and \( p \) = probability of success. However, a project may be successful only if it is selected in the solution. To model this, we use an IF statement and randomly assign 1 or 0 to cells B15:F15, depending on whether the projects are selected or not. For example, the formula in cell B15 is \( =IF(B14=1,\PsiBinomial(1,B7),0) \). Similarly, the returns in cells B16:F16 also depend on whether the projects are selected. Thus, the formula in cell B16 is \( =IF(B15=1,\PsiLogNormal(B5,B6),0) \).

To set up the optimization model, select the range B14:F14, click on the Decisions button in the Optimization Model group in the Analytic Solver Platform ribbon, and select Normal. Also, set these variables as binary. Select the constraint function cells G17:G18, and click on the Constraints button; choose Normal and then \( \leq \). Add the right-hand sides of the constraints to the Add Constraint dialog by referencing cells G9:G10. Finally, select the objective cell G16, and click on the Objective button. Select Max category and choose Expected from the list. Because we have a non-smooth problem, we need to use Evolutionary Solver. This can be done by either choosing Standard Evolutionary Engine from the Engine tab in the Analytic Solver Platform task pane, or selecting it in the Solver Parameters dialog from the Excel Add-Ins tab. Click the Optimize button in the Solve Action group in the ribbon, and Analytic Solver Platform will find the optimal solution.

You can view both the optimization and simulation models by clicking the Model button on the Analytic Solver Platform ribbon. This displays the “task pane” on the right side of the spreadsheet. You may also use the task pane to create your model. The solution in Figure B.22 found by using Evolutionary Solver is to choose projects 1, 3, and 4. The actual values of success and return only represent a single simulation trial and not the mean values.

Figure B.23 shows the simulation results for the output cell G16. The mean return is $378,854. Note that the frequency distribution looks like it is composed of several different distributions, and indeed it is. This is because of the discrete nature of the success of individual projects. If all are not successful, the distribution of the return will be different. This illustrates the nature of the risk in making the decision to choose the optimal set of projects.
Supplementary Chapter B  Optimization Models with Uncertainty

### Table B.1

Hahn Engineering Project Selection Data with Uncertainty

<table>
<thead>
<tr>
<th>Available Resources</th>
<th>Project 1</th>
<th>Project 2</th>
<th>Project 3</th>
<th>Project 4</th>
<th>Project 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean return (NPV)</td>
<td>$180,000</td>
<td>$220,000</td>
<td>$150,000</td>
<td>$140,000</td>
<td>$200,000</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>$15,000</td>
<td>$20,000</td>
<td>$10,000</td>
<td>$10,000</td>
<td>$25,000</td>
</tr>
<tr>
<td>Probability of success</td>
<td>0.9</td>
<td>0.9</td>
<td>0.7</td>
<td>0.8</td>
<td>0.5</td>
</tr>
<tr>
<td>Cash requirements</td>
<td>$55,000</td>
<td>$83,000</td>
<td>$24,000</td>
<td>$49,000</td>
<td>$61,000</td>
</tr>
<tr>
<td>Personnel requirements</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

**Figure B.22**

Uncertain Project-Selection Model

**Figure B.23**

Simulation Results for Project-Selection Model
Supplementary Chapter B  Optimization Models with Uncertainty

Key Terms

<table>
<thead>
<tr>
<th>Chance constraint</th>
<th>Service level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional value at risk (CVaR)</td>
<td>Value at risk (VaR)</td>
</tr>
<tr>
<td>Multiple parameterized simulation</td>
<td></td>
</tr>
</tbody>
</table>

Problems and Exercises

1. In the *Sklenka Skis* model, find a solution that provides a probability that overtime will be needed no more than 20% of the time. Verify your solution by examining the frequency chart from a Monte Carlo simulation and compare the VaR and CVaR types of chance constraints.

2. In Problem 3 of Chapter 13 (*Burger Office Equipment*), suppose that the amount of labor required per desk is uncertain. Assume that the amount of labor required for a standard desk is triangular with parameters (9, 10, 12), and the amount of labor required for a deluxe desk is triangular with parameters (14, 16, 21). If the company wishes to limit the probability of requiring overtime beyond the 400 hours available to at most 20%, how many desks of each type should they produce?

3. In the economic order quantity model discussed in this chapter, find the reorder point to support service levels of 90%, 80%, and 75%.

4. The annual demand for the bolt-nut package for the aircraft manufacturer Excel database Purchase Orders (see Example 1.3 in Chapter 1) is 60,000. The fixed cost of placing an order is $50, and the unit cost of the item is $3.95. The company uses an annual carrying charge of 15%.
   a. Use the *Economic Order Quantity Model* spreadsheet and *Solver* to find the optimal order quantity.
   b. If the annual demand is lognormally distributed with a mean of 60,000 and a standard deviation of 4500, what is the distribution of lead-time demand, if the lead time is one week?
   c. What reorder point should be used to ensure a probability of running out of stock of at most 10%?

5. For the Innis Investments example in Chapter 14, suppose that the investment returns are uncertain. Assume that each can be modeled as a lognormal distribution with a mean equal to the expected return in the example and a standard deviation equal to 10% of the mean.

6. For the optimal solution shown in Figure 14.15, what is the probability that the target return of 5% will be achieved?

7. For the Rosenberg Land Development problem (Problem 2 in Chapter 14), suppose that the construction costs are uncertain. Specifically, assume that the distribution of construction costs is normally distributed, with the mean values as given, and standard deviations equal to 15% of the mean. Using the optimal solution to the linear optimization model, find the probability of exceeding the budget after construction is started.

8. Develop a Monte Carlo simulation model for the gasoline mini-mart situation described in Problem 12 of Chapter 11. Use the *IntUniform* distribution in *Analytic Solver Platform* to model the demand and find the distribution of profit for an order quantity of 15. Use multiple parameterized simulation to identify the best order quantity between 10 and 30 to maximize profit.

9. A sporting goods store orders ski jackets in the summer before the winter season. Each jacket costs $80 and sells for $200. Any not sold by March are discounted 75%. Demand depends on the weather. The distribution of demand is triangular (but must involve whole numbers) with a minimum of 40, maximum of 150, and most likely value of 80. How many jackets should the retailer order?

10. Midwestern Hardware must decide how many snow shovels to order for the coming snow season. Each shovel costs $15.00 and is sold for $29.95. No
inventory is carried from one snow season to the next. Shovels unsold after February are sold at a discount price of $10.00. Past data indicate that sales are highly dependent on the severity of the winter season. Past seasons have been classified as mild or harsh, and the forecast calls for a 70% chance of a harsh winter. The following distribution of regular price demand has been tabulated:

<table>
<thead>
<tr>
<th>No. of Shovels</th>
<th>Probability</th>
<th>No. of Shovels</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.5</td>
<td>1,500</td>
<td>0.2</td>
</tr>
<tr>
<td>300</td>
<td>0.4</td>
<td>2,500</td>
<td>0.4</td>
</tr>
<tr>
<td>350</td>
<td>0.1</td>
<td>3,000</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Shovels must be ordered from the manufacturer in lots of 200. Develop a Monte Carlo simulation model to find the profit for any order quantity and use multiple parameterized simulation to identify the best order quantity.

11. Bev’s Bakery specializes in sourdough bread. Early each morning, Bev must decide how many loaves to bake for the day. Each loaf costs $0.75 to make and sells for $2.85. Bread left over at the end of the day can be sold the next for $0.50. Past data indicate that demand is distributed as follows:

<table>
<thead>
<tr>
<th>Number of Loaves</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.05</td>
</tr>
<tr>
<td>16</td>
<td>0.05</td>
</tr>
<tr>
<td>17</td>
<td>0.10</td>
</tr>
<tr>
<td>18</td>
<td>0.10</td>
</tr>
<tr>
<td>19</td>
<td>0.20</td>
</tr>
<tr>
<td>20</td>
<td>0.35</td>
</tr>
<tr>
<td>21</td>
<td>0.10</td>
</tr>
<tr>
<td>22</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Develop a Monte Carlo simulation model to find the profit for baking any quantity of bread and use multiple parameterized simulation to identify the best number to bake.

12. Using the simulation model for Koehler Vision Associates (KVA) in Problem 23 of Chapter 12, determine the best level of overbooking to maximize the net profit (revenue less overbooking costs).

13. For the stock broker problem described in Problem 10 in Chapter 11, assume that among the qualified clients, half will invest between $2,000 and $10,000, 25% will invest between $10,000 and $25,000, 15% will invest between $25,000 and $50,000, and the remainder will invest between $50,000 and $100,000, each uniformly distributed. Using the same commission schedule, how many calls per month must the broker make each month to have at least a 75% chance of making at least $5,000?

14. For the nonprofit ballet company in Problem 11 of Chapter 11, assume following percentages of donors and gift levels:

<table>
<thead>
<tr>
<th>Gift Level</th>
<th>Amount</th>
<th>Number of Gifts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefactor</td>
<td>$10,000</td>
<td>1–3</td>
</tr>
<tr>
<td>Philanthropist</td>
<td>$5,000</td>
<td>3–7</td>
</tr>
<tr>
<td>Producer’s Circle</td>
<td>$1,000</td>
<td>16–25</td>
</tr>
<tr>
<td>Director’s Circle</td>
<td>$500</td>
<td>31–40</td>
</tr>
<tr>
<td>Principal</td>
<td>$100</td>
<td>5–7% of solicitations</td>
</tr>
<tr>
<td>Soloist</td>
<td>$50</td>
<td>5–7% of solicitations</td>
</tr>
</tbody>
</table>

The company has set a financial goal of $150,000. How many prospective donors must they contact for donations at the $100 level or below to have a 95% chance of meeting this goal? Assume that the number of gifts at each level follow a discrete uniform distribution or a uniform distribution for the percentage of solicitations at the $100 and $50 levels.

15. For the Innis Investments example in Chapter 14, suppose that the investment returns are uncertain. Assume that each can be modeled as a lognormal distribution with a mean equal to the expected return in the example and a standard deviation equal to 10% of the mean. Use Analytic Solver Platform to maximize the expected return and find the distribution of the expected return.

16. For the medical device company problem (Problem 12 in Chapter 15), suppose that the net present values of the projects are uncertain. The best estimates of the mean values are given in the problem statement in Chapter 15. Assume that the distributions of NPV are lognormally distributed with standard deviations equal to 15% of the
Means. In addition, the probability that any project will be successfully completed is estimated to be 0.8.

a. Use Analytic Solver Platform to find the best set of projects to select.

b. Conduct sensitivity analyses on the probability of successful completion and summarize your results.

17. For the software-support-project problem (Problem 13 in Chapter 15), suppose that the transfer prices are uncertain. Assume that each has a triangular distribution with the most likely value equal to the value given in the problem, a minimum value that is 20% less, and a maximum value that is 12% higher. Use Analytic Solver Platform to find the best set of projects to select.

Case: Performance Lawn Equipment

After solving the model in the case in Chapter 13, Elizabeth Burke discovered that the times required in the assembly and painting processes, which are heavily labor intensive, vary from their estimate used in that model. Specifically, the production rate for mower housings in assembly have a triangular distribution with a minimum value of 0.05, most likely value of 0.05, and maximum value of 0.07 hours; tractor housings are also triangular with parameters 0.05, 0.06, and 0.09; painting time for mower housings is uniform between 0.032 and 0.049 hours; and painting time for tractor housings is also uniform between 0.055 and 0.075 hours.

In addition, she noted that the amount of sheet metal used is often more than the amount required because of supplier defects, which lead to scrap. Specifically, the amount of sheet metal needed for mower housings is triangular with parameters 1.2, 1.3, and 1.5 square feet; and the amount of sheet metal for tractor housings is triangular with parameters 1.8, 1.10, and 1.15 square feet.

Ms. Burke has asked you to revise the production plan that you recommended in the case in Chapter 13. Use whatever approaches and analyses you feel are necessary to provide a rational plan, and summarize your results in a report to Ms. Burke.