Section 8.5 Solving Equations Containing Rational Expressions

**Before Class:**

- Read the objectives on page 521.
- Read the **Helpful Hint** boxes on pages 521, 522, and 524.
- Complete the exercises:
  1. To solve equations containing rational expressions, use the ____________________________ of equality to clear the equation of fractions by ____________________________ both sides of the equation by the LCD.
  2. What is the last step to solving an equation containing rational expressions?

**During Class:**

- Write your class notes. Neatly write down all examples shown as well as key terms or phrases with definitions. If not applicable or if you were absent, watch the Lecture Series (DVD) for this section and do the same (write down the examples shown as well as key terms or phrases). Insert more paper as needed.

<table>
<thead>
<tr>
<th>Class Notes/Examples</th>
<th>Your Notes</th>
</tr>
</thead>
</table>

*Answers:* 1) multiplication property; multiplying 2) Check the solution in the original equation.
## Section 8.5 Solving Equations Containing Rational Expressions

<table>
<thead>
<tr>
<th>Class Notes (continued)</th>
<th>Your Notes</th>
</tr>
</thead>
</table>

(Insert additional paper as needed.)
Section 8.5 Solving Equations Containing Rational Expressions

Practice:

☐ Complete any incomplete exercises below. Check and correct your work using the answers and references at the end of this section.

Review this example:

1. Solve: \( \frac{t - 4}{2} - \frac{t - 3}{9} = \frac{5}{18} \)

   The LCD of denominators 2, 9, and 18 is 18, so we multiply both sides of the equation by 18.

   \[
   18 \left( \frac{t - 4}{2} - \frac{t - 3}{9} \right) = 18 \left( \frac{5}{18} \right) \\
   18 \left( \frac{t - 4}{2} \right) - 18 \left( \frac{t - 3}{9} \right) = 18 \left( \frac{5}{18} \right) \\
   9(t - 4) - 2(t - 3) = 5 \\
   9t - 36 - 2t + 6 = 5 \\
   7t - 30 = 5 \\
   7t = 35 \\
   t = 5
   \]

   Check: \( \frac{t - 4}{2} - \frac{t - 3}{9} = \frac{5}{18} \)

   \[
   \frac{5 - 4}{2} - \frac{5 - 3}{9} = \frac{5}{18} \\
   \frac{1}{2} - \frac{2}{9} = \frac{5}{18} \\
   \frac{2 - 4}{9} = \frac{5}{18} \\
   True \quad \frac{5}{18} = \frac{5}{18}
   \]

   The solution is \( t = 5 \).

Your turn:

2. Solve the equation and check each solution.

   \( \frac{x - 3}{5} + \frac{x - 2}{2} = \frac{1}{2} \)

Review this example:

3. Solve: \[\frac{4x}{x^2 + x - 30} + \frac{2}{x - 5} = \frac{1}{x + 6}\]

The denominator \(x^2 + x - 30\) factors as \((x + 6)(x - 5)\). The LCD is then \((x + 6)(x - 5)\).

\[
\begin{align*}
(x + 6)(x - 5) \left(\frac{4x}{x^2 + x - 30} + \frac{2}{x - 5}\right) &= (x + 6)(x - 5) \left(\frac{1}{x + 6}\right) \\
(x + 6)(x - 5) \cdot \frac{4x}{x^2 + x - 30} + (x + 6)(x - 5) \cdot \frac{2}{x - 5} &= (x + 6)(x - 5) \cdot \frac{1}{x + 6} \\
4x + 2(x + 6) &= x - 5 \\
4x + 2x + 12 &= x - 5 \\
6x + 12 &= x - 5 \\
5x &= -17 \\
x &= -\frac{17}{5}
\end{align*}
\]

Check by replacing \(x\) with \(-\frac{17}{5}\) in the original equation.

The solution is \(-\frac{17}{5}\).

Your turn:

4. Solve: \[\frac{4r - 4}{r^2 + 5r - 14} + \frac{2}{r + 7} = \frac{1}{r - 2}\]
**Review this example:**

5. Solve: \( x + \frac{14}{x-2} = \frac{7x}{x-2} + 1 \)

Notice the denominators in this equation. We can see that 2 can’t be a solution. The LCD is \( x - 2 \), so we multiply both sides of the equation by \( x - 2 \).

\[
(x - 2) \left( x + \frac{14}{x-2} \right) = (x - 2) \left( \frac{7x}{x-2} + 1 \right)
\]

\[
(x - 2)(x) + (x - 2) \left( \frac{14}{x-2} \right) = (x - 2) \left( \frac{7x}{x-2} \right) + (x - 2)(1)
\]

\[
x^2 - 2x + 14 = 7x + x - 2
\]

\[
x^2 - 2x + 14 = 8x - 2
\]

\[
x^2 - 10x + 16 = 0
\]

\[
(x - 8)(x - 2) = 0
\]

\[
x - 8 = 0 \quad \text{or} \quad x - 2 = 0
\]

\[
x = 8 \quad \text{or} \quad x = 2
\]

2 can’t be a solution of the original equation. So we need only replace \( x \) with 8 in the original equation. We find that 8 is a solution; the only solution is 8.

**Your turn:**

6. Solve: \( \frac{t}{t-4} = \frac{t+4}{6} \)

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**Review this example:**

7. Solve \( \frac{1}{a} + \frac{1}{b} = \frac{1}{x} \) for \( x \).

The LCD is \( abx \), so we multiply both sides by \( abx \).

\[
abx \left( \frac{1}{a} + \frac{1}{b} \right) = abx \left( \frac{1}{x} \right)
\]

\[
abx \left( \frac{1}{a} \right) + abx \left( \frac{1}{b} \right) = abx \left( \frac{1}{x} \right)
\]

\[
bx + ax = ab
\]

\[
x(b+a) = ab
\]

\[
\frac{x(b+a)}{b+a} = \frac{ab}{b+a}
\]

\[
x = \frac{ab}{b+a}
\]

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**Your turn:**

8. Solve \( T = \frac{2U}{B+E} \) for \( B \).
Section 8.5 Solving Equations Containing Rational Expressions

<table>
<thead>
<tr>
<th>Answer</th>
<th>Text Ref</th>
<th>Video Ref</th>
<th>Answer</th>
<th>Text Ref</th>
<th>Video Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>Ex 2, p. 521</td>
<td>5</td>
<td>8</td>
<td>Ex 6, p. 524</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>Sec 8.5, Ex 11</td>
<td>6</td>
<td>8, −2</td>
<td>Sec 8.5, Ex 35</td>
</tr>
<tr>
<td>3</td>
<td>−17/5</td>
<td>Ex 4, p. 523</td>
<td>7</td>
<td>x = \frac{ab}{b+a}</td>
<td>Ex 7, p. 524</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>Sec 8.5, Ex 39</td>
<td>8</td>
<td>B = \frac{2U - TE}{T}</td>
<td>Sec 8.5, Ex 45</td>
</tr>
</tbody>
</table>

☐ Next, insert your homework. Make sure you attempt all exercises asked of you and show all work, as in the exercises above. Check your answers if possible. Clearly mark any exercises you were unable to correctly complete so that you may ask questions later. DO NOT ERASE YOUR INCORRECT WORK. THIS IS HOW WE UNDERSTAND AND EXPLAIN TO YOU YOUR ERRORS.