Before Class:

☐ Read the objectives on page 540.

☐ Read the Helpful Hint box on page 545.

☐ Complete the exercises:

1. $y$ varies ______________________ as $x$ if there is a nonzero constant $k$ such that $y = kx$.

2. $y$ varies ______________________ as $x$ if there is a nonzero constant $k$ such that $y = \frac{k}{x}$.

During Class:

☐ Write your class notes. Neatly write down all examples shown as well as key terms or phrases with definitions. If not applicable or if you were absent, watch the Lecture Series (DVD) for this section and do the same (write down the examples shown as well as key terms or phrases). Insert more paper as needed.

<table>
<thead>
<tr>
<th>Class Notes/Examples</th>
<th>Your Notes</th>
</tr>
</thead>
</table>

**Answers:** 1) directly 2) inversely
Section 8.7 Variation and Problem Solving

<table>
<thead>
<tr>
<th>Class Notes (continued)</th>
<th>Your Notes</th>
</tr>
</thead>
</table>

(Insert additional paper as needed.)
Section 8.7 Variation and Problem Solving

Practice:

☐ Complete the Vocabulary and Readiness Check on page 547.

☐ Next, complete any incomplete exercises below. Check and correct your work using the answers and references at the end of this section.

Review this example:
1. Write a direct variation equation of the form \( y = kx \) that satisfies the given ordered pairs.

\[
\begin{array}{c|cccc}
 x & 2 & 9 & 1.5 & -1 \\
 y & 6 & 27 & 4.5 & -3 \\
\end{array}
\]

We are given that there is a direct variation relationship between \( x \) and \( y \). This means that \( y = kx \). To find \( k \), substitute one given ordered pair into this equation and solve for \( k \). Use \((2,6)\).

\[
y = kx \\
6 = k \cdot 2 \\
\frac{6}{2} = k \\
3 = k
\]

Since \( k = 3 \), we have the equation \( y = 3x \). Check: See that each given \( y \) is 3 times the given \( x \).

Your turn:
2. Write a direct variation equation, \( y = kx \), that satisfies the ordered pairs in the table.

\[
\begin{array}{c|cccc}
 x & -2 & 2 & 4 & 5 \\
 y & -12 & 12 & 24 & 30 \\
\end{array}
\]

Review this example:
3. Suppose that \( y \) varies directly as \( x \). If \( y = 17 \) when \( x = 34 \), find the constant of variation and the direct variation equation. Then find \( y \) when \( x = 12 \).

We know the relationship is of the form \( y = kx \). Let \( y = 17 \) and \( x = 34 \) and solve for \( k \).

\[
17 = k \cdot 34 \\
\frac{17}{34} = \frac{k \cdot 34}{34} \\
\frac{1}{2} = k \\
Thus the equation is \( y = \frac{1}{2}x \).
\]

To find \( y \) when \( x = 12 \), replace \( x \) with 12 in

\[
y = \frac{1}{2}x \\
y = \frac{1}{2} \cdot 12 = 6
\]

Thus, when \( x \) is 12, \( y \) is 6.

Your turn:
4. \( y \) varies directly as \( x \). If \( y = 20 \) when \( x = 5 \), find \( y \) when \( x = 10 \).
Section 8.7 Variation and Problem Solving

**Review this example:**

5. Write an inverse variation equation of the form \( y = \frac{k}{x} \) that satisfies the ordered pairs in the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>4</th>
<th>( \frac{1}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>6</td>
<td>3</td>
<td>24</td>
</tr>
</tbody>
</table>

We know that \( y = \frac{k}{x} \). To find \( k \), choose one given ordered pair and substitute the values into the equation. Use \((2,6)\).

\[
y = \frac{k}{x} \\
6 = \frac{k}{2} \\
2 \cdot 6 = 2 \cdot \frac{k}{2} \\
12 = k
\]

Since \( k = 12 \), we have the equation \( y = \frac{12}{x} \).

**Your turn:**

6. Write an inverse variation equation, \( y = \frac{k}{x} \), that satisfies the ordered pairs in the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>−7</th>
<th>3.5</th>
<th>−2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>7</td>
<td>−1</td>
<td>2</td>
<td>−3.5</td>
</tr>
</tbody>
</table>

**Review this example:**

7. Suppose that \( y \) varies inversely as \( x \). If \( y = 0.02 \) when \( x = 75 \), find the constant of variation and the inverse variation equation. Then find \( y \) when \( x \) is 30.

Since \( y \) varies inversely as \( x \), the constant of variation may be found by simply finding the product of the given \( x \) and \( y \).

\[
k = xy = 75 \times 0.02 = 1.5
\]

Thus, the equation is \( y = \frac{1.5}{x} \). To find \( y \) when \( x = 30 \), replace \( x \) with 30 in \( y = \frac{1.5}{x} \).

\[
y = \frac{1.5}{30} \\
y = \frac{1.5}{30} \\
y = 0.05
\]

Thus, when \( x \) is 30, \( y \) is 0.05.

**Your turn:**

8. \( y \) varies inversely as \( x \). If \( y = 5 \) when \( x = 60 \), find \( y \) when \( x \) is 100.
Review this example:

9. The weight of a body \( w \) varies inversely with the square of its distance from the center of Earth, \( d \). If a person weighs 160 pounds on the surface of Earth, what is the person’s weight 200 miles above the surface? (Assume that the radius of Earth is 4000 miles.)

UNDERSTAND. Read and reread the problem.

TRANSLATE. Since weight, \( w \), varies inversely with the square of its distance from the center of Earth, \( d \), we have \( w = \frac{k}{d^2} \).

SOLVE. First find \( k \). Use the fact that the person weighs 160 pounds on Earth’s surface, which is a distance of 4000 miles from Earth’s center.

\[
160 = \frac{k}{(4000)^2}
\]

or \( 2,560,000,000 = k \).

Thus \( w = \frac{2,560,000,000}{d^2} \).

Since we want to know the person’s weight 200 miles above the Earth’s surface, let \( d = 4200 \) and find \( w \).

\[
w = \frac{2,560,000,000}{(4200)^2}
\]

or \( w = 145 \).

INTERPRET.

Check: Your answer is reasonable since the farther a person is from Earth, the less the person weighs.

State: 200 miles above Earth’s surface, a 160-pound person weighs approximately 145 pounds.

Your turn:

10. The distance a spring stretches varies directly with the weight attached to the spring. If a 60-pound weight stretches the spring 4 inches, find the distance that an 80-pound weight stretches the spring.
### Section 8.7 Variation and Problem Solving

<table>
<thead>
<tr>
<th></th>
<th>Answer</th>
<th>Text Ref</th>
<th>Video Ref</th>
<th></th>
<th>Answer</th>
<th>Text Ref</th>
<th>Video Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$y = 3x$</td>
<td>Ex 1, p. 541</td>
<td></td>
<td>6</td>
<td>$y = \frac{7}{x}$</td>
<td></td>
<td>Sec 8.7, Ex 9</td>
</tr>
<tr>
<td>2</td>
<td>$y = 6x$</td>
<td>Sec 8.7, Ex 3</td>
<td></td>
<td>7</td>
<td>$y = 0.05$</td>
<td>Ex 5, p. 545</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$y = 6$</td>
<td>Ex 2, p. 541</td>
<td></td>
<td>8</td>
<td>$y = 3$</td>
<td></td>
<td>Sec 8.7, Ex 25</td>
</tr>
<tr>
<td>4</td>
<td>$y = 40$</td>
<td>Sec 8.7, Ex 23</td>
<td></td>
<td>9</td>
<td>145 lb</td>
<td>Ex 7, p. 546</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$y = \frac{12}{x}$</td>
<td>Ex 4, p. 544</td>
<td></td>
<td>10</td>
<td>$\frac{5\frac{1}{3}}{3}$ in.</td>
<td></td>
<td>Sec 8.7, Ex 35</td>
</tr>
</tbody>
</table>

☐ **Next, insert your homework.** Make sure you attempt all exercises asked of you and show all work, as in the exercises above. Check your answers if possible. Clearly mark any exercises you were unable to correctly complete so that you may ask questions later. DO NOT ERASE YOUR INCORRECT WORK. THIS IS HOW WE UNDERSTAND AND EXPLAIN TO YOU YOUR ERRORS.