Before Class:

☐ Read the objectives on page 688.

☐ Read the Helpful Hint box on page 691.

☐ Complete the exercises:

1. A __________________________ is an ordered arrangement of items in which no item is used more than once and the order of arrangement makes a difference.

2. For $n > 0$, the notation $n!$ is the product of what?

3. What does the notation $_nP_r$ mean?

During Class:

☐ Write your class notes. Neatly write down all examples shown as well as key terms or phrases with definitions. If not applicable or if you were absent, watch the Lecture Series (DVD) for this section and do the same (write down the examples shown as well as key terms or phrases). Insert more paper as needed.

<table>
<thead>
<tr>
<th>Class Notes/Examples</th>
<th>Your Notes</th>
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*Answers: 1) permutation  2) all positive integers from $n$ down through 1  3) the number of permutations of $n$ things taken $r$ at a time*
### Appendix D Permutations

<table>
<thead>
<tr>
<th>Class Notes (continued)</th>
<th>Your Notes</th>
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(Insert additional paper as needed.)
Practice:

☐ Complete any incomplete exercises below. Check and correct your work using the answers and references at the end of this section.

Review this example:

1. You need to arrange seven of your favorite books along a small shelf. How many different ways can you arrange the books, assuming that the order of the books makes a difference to you?

You may choose any of the seven books for the first position on the shelf. This leaves six choices for second position. After the first two positions are filled, there are five books to choose from for the third position, four choices left for the fourth position, three choices left for the fifth position, then two choices for the sixth position, and only one choice for the last position.

We use the Fundamental Counting Principle to find the number of ways you can arrange the seven books along the shelf. Multiply the choices:

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

Thus, you can arrange the books in 5040 ways.

Your turn:

2. Six performers are to present their comedy acts on a weekend evening at a comedy club. How many different ways are there to schedule their appearances? Use the Fundamental Counting Principle to solve.

Review this example:

3. Evaluate the following factorial expressions without using the factorial key on your calculator.

   a. \( \frac{8!}{5!} \)  
   b. \( \frac{26!}{21!} \)  
   c. \( \frac{500!}{499!} \)

   a. \( \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 8 \cdot 7 \cdot 6 = 336 \)
   
   b. \( \frac{26!}{21!} = \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21!}{21!} = 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 = 7,893,600 \)
   
   c. \( \frac{500!}{499!} = \frac{500 \cdot 499!}{499!} = 500 \)

Your turn:

4. Evaluate each factorial expression.

   a. \( \frac{9!}{6!} \)
   
   b. \( \frac{7!}{(7-2)!} \)
**Appendix D Permutations**

**Review this example:**
5. You and 19 of your friends have decided to form an Internet marketing consulting firm. The group needs to choose three officers—a CEO, an operating manager, and a treasurer. In how many ways can those offices be filled?

Your group is choosing \( r = 3 \) officers from a group of \( n = 20 \) people. The order in which the officers are chosen matters because the officers each have different responsibilities. Thus, we are looking for the number of permutations of 20 things taken 3 at a time.

\[
\begin{align*}
\text{20P}_3 &= \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20\cdot 19\cdot 18\cdot 17!}{17!} \\
&= \frac{20\cdot 19\cdot 18}{6840}
\end{align*}
\]

There are 6840 different ways to fill the three offices.

**Your turn:**
6. For a segment of a radio show, a disc jockey can play 7 records. If there are 13 records to select from, in how many ways can the program for this segment be arranged? Use the formula for \( _nP_r \).

7. In how many distinct ways can the letters of the word MISSISSIPPI be arranged?

The word contains 11 letters \((n = 11)\), where four Is are identical \((p = 4)\), four Ss are identical \((q = 4)\), and 2 Ps are identical \((r = 2)\). The number of distinct permutations is

\[
\frac{n!}{p!q!r!} = \frac{11!}{4!4!2!} = 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4! \cdot 4!\cdot 3\cdot 2\cdot 1\cdot 2\cdot 1 = 34,650
\]

There are 34,650 distinct ways the letters in the word MISSISSIPPI can be arranged.

**Your turn:**
8. In how many distinct ways can the letters of the word DALLAS be arranged? Use the formula for the number of permutations of duplicate items to solve.

<table>
<thead>
<tr>
<th>Answer</th>
<th>Text Ref</th>
<th>Video Ref</th>
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<tbody>
<tr>
<td>1</td>
<td>5040</td>
<td>Ex 2, p. 689</td>
</tr>
<tr>
<td>2</td>
<td>720</td>
<td>App D, Ex 1</td>
</tr>
<tr>
<td>3</td>
<td>a. 336</td>
<td>Ex 3, p. 690</td>
</tr>
<tr>
<td></td>
<td>b. 7,893,600</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c. 500</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a. 504</td>
<td>App D, Ex 13, 29</td>
</tr>
<tr>
<td></td>
<td>b. 42</td>
<td></td>
</tr>
</tbody>
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**Next, insert your homework.** Make sure you attempt all exercises asked of you and show all work, as in the exercises above. Check your answers if possible. Clearly mark any exercises you were unable to correctly complete so that you may ask questions later. **DO NOT ERASE YOUR INCORRECT WORK. THIS IS HOW WE UNDERSTAND AND EXPLAIN TO YOU YOUR ERRORS.**